

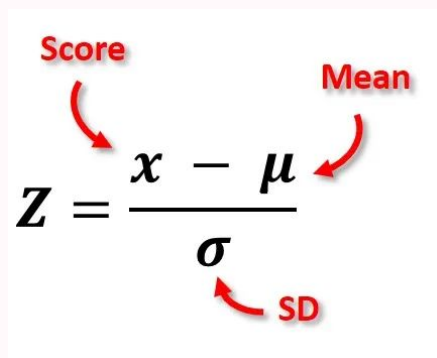
# Official ASA 2025 ELO Documentation

## Tabulation

The ASA rubric consists of **4 major sections** : Musical Composition, Vocal Execution, Visual Execution, and South Asian Representation. At each competition, judges will assign scores to teams in each category, which will subsequently be converted to a 100-point scale for convenience.

## Normalization

Each team's raw score per judge is normalized by computing its z-score, whose formula is given below. To do this, the average score for each judge as well as the standard deviation, or spread from the average, is calculated. For each team's raw score per judge, the average score is subtracted and then divided by the standard deviation of the respective judge. Next, the mean of the normalized scores for each team is computed. These mean values will be used as the main unbiased data point to account for outliers in judge scoring and act as a synthetic "ground-truth" judge. At the end of each competition, each team will now have a set of normalized scores for each judge along with the mean of aforementioned scores at that competition, the latter of which will be used for ELO rating calculation.


$$Z = \frac{x - \mu}{\sigma}$$

The diagram shows the z-score formula with red arrows pointing to the variables: 'Score' points to  $x$ , 'Mean' points to  $\mu$ , and 'SD' (Standard Deviation) points to  $\sigma$ .

## ELO Ratings

Team ratings will be determined by a generalized approach to the ELO rating system. ELO is a renowned system that is a way to keep track of how good a player is at a game. Essentially, a team's ELO rating is represented by a number that either increases or decreases depending on how many other teams it outperforms. Teams with lower ELO ratings will win more points from higher-rated teams they beat and conversely, teams with high ELO ratings will win fewer points from lower-rated teams they beat. Traditionally, ELO is used widely for 1-on-1 matchups as in chess tournaments, for example, but we apply a generalized ELO system for multi-team competitions. Note that ELO will only be used for selection to A3 while scoring/ranking at individual bid competitions will remain the same.

Following sections document the necessary steps for generating ELO ratings and discuss the system's goals.

## Rating Initialization

At the beginning of the ASA competition circuit, every team begins with the same arbitrary rating of 1500, which is reflected by the variable  $R_i$ .

## Expected Scores

Expected scores,  $E_A$ , are used to predict the outcome of a competition, or essentially how likely a team is to win. Standard ELO can only compare two players at a time, so let's instead think of a multi-team a cappella competition as a series of one-on-one matches. For example, if there are three teams, we have three separate matches (A vs. B, A vs. C, and B vs. C). In general, if there are  $N$  teams, there are  $N(N-1)/2$  individual matches.

To figure this out, we'll use the regular ELO method to predict the scores for each pair of teams using a logistic function as depicted below. Then, we'll add up the expected scores for each team from all their matches to find their overall expected score. Finally, we'll adjust the scores so that they add up to 1 for all teams. This helps us see the scores as probabilities, like we do in the standard ELO system.

$$E_A = \frac{\sum_{1 \leq i \leq N, i \neq A} \frac{1}{1 + 10^{(R_i - R_A)/D}}}{N(N - 1)/2}$$

## Actual Scores

For each competition, the results of the matchups are observed. In standard ELO, the results follow a binary representation where one player is assigned a score of 1 if they win and 0 if they lose. For multi-team a cappella competitions, the results are encoded with a score in a similar manner. The scores themselves abide by certain conditions: 1) They must be monotonically decreasing, 2) Last place must have a score of 0, and 3) The scores must sum to 1 across all teams.

For the given "synthetic judge", find it's minimum normalized score. The synthetic judge's minimum score is then subtracted from each team's normalized score to get a delta score, depicting how much better each team performed vs. the lowest-scoring team. The sum of all teams' delta scores is then calculated. This delta score is then divided by the overall sum. The final resulting scores are the actual scores assigned to each team, retaining the weighting of how much higher each team scores vs. the lowest-scoring team.

## Rating Update

$$R'_A = R_A + K(N - 1)(S_A - E_A)$$

The new ratings for each team,  $R'_A$ , are calculated by simply adding the current team's rating,  $R_A$ , with the product of a constant  $K$  factor and the difference in expected and actual scores. The  $K$  factor in the ELO rating system determines how much a team's rating changes after a competition. A higher  $K$  factor means bigger rating swings, making it useful for new or rapidly improving teams. Established teams might use a lower  $K$  factor, resulting in more gradual changes.

At a competition, the above calculations for new team ratings are computed for each team. For newer teams, or those who have been to either 0 or 1 competition, the  $K$  factor is set at an arbitrary value of 20. For teams that have already competed at one or more competitions, the  $K$  factor is set to 16, or 80% of the original  $K$  factor. The varying  $K$  factor assignment is made to simulate convergence, or the rating system's way of making sure a team's rating changes become steadier and more reflective of their actual skill as they compete at more competitions.

## Removing Time-Based Factors

ELO is inherently a time based system, as updates are made based on the order in which events occur. With a limited number of competitions occurring throughout the season, teams that face equally rated opponents early on may be at a slight disadvantage, simply due to a lack of prior competition. To address this, we now calculate ELO rankings using every possible permutation of dates in the competition schedule. For instance, with nine competitions, and on five unique competition dates in the 2025 season, we evaluate all 5! (120) permutations and derive each team's final ELO rating by averaging its rating across all permutations. This approach eliminates the unintended chronological bias and provides a more accurate reflection of team performance.

## Advantages

The use of the ELO rating system provides several key advantages. One of the foremost advantages is that the strength of schedule for each team is taken into account. Consecutively winning against lower-rated teams will not gain a team as many points as winning against many higher-rated teams will. Also, competing fewer times than other teams does not put a team at a major disadvantage and teams that do compete a lot aren't rewarded by just competing. Finally, teams that consistently perform at a high level over the season are rewarded.

## Qualification

Once a competition has ended, a list of updated ratings for each competing team is produced and then sorted by rating. After all bid competitions have ended, all competitive teams and their final ratings are compiled together and ranked accordingly. The top  $n$  highest-rated teams are then selected as the teams that have successfully qualified for A3.

Rank	Team	Initial Rating	Final Rating
1	Team A	1500	1530
2	Team B	1500	1521
3	Team C	1500	1519
4	Team D	1500	1516
5	Team E	1500	1505
6	Team F	1500	1502
7	Team G	1500	1489